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Negative Parity 70-plet Baryon Masses in the $1/N_c$ Expansion

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Abstract

The masses of the negative parity $SU(6)$ 70-plet baryons are analyzed in the $1/N_c$ expansion to order $1/N_c$ and to first order in $SU(3)$ breaking. At this level of precision there are twenty predictions. Among them there are the well known Gell-Mann Okubo and equal spacing relations, and four new relations involving $SU(3)$ breaking splittings in different $SU(3)$ multiplets. Although the breaking of $SU(6)$ symmetry occurs at zeroth order in $1/N_c$, it turns out to be small. The dominant source of the breaking is the hyperfine interaction which is of order $1/N_c$. The spin-orbit interaction, of zeroth order in $1/N_c$, is entirely fixed by the splitting between the singlet states $\Lambda(1405)$ and $\Lambda(1520)$, and the spin-orbit puzzle is solved by the presence of other zeroth order operators involving flavor exchange.

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I. INTRODUCTION

The understanding of baryons and their excitations from QCD still represents a wide open chapter in strong interaction physics. This is naturally so as baryons are the sector of the strong interactions where the non-perturbative QCD dynamics is likely to be most difficult to sort out. With some exceptions, the current understanding of the baryon sector is largely based on data of different sorts and its analyses by means of models, most prominently the constituent quark model in its different versions [1]. One exception are the ground state baryons, namely the spin 1/2 octet and spin 3/2 decouplet, where definite progress in relating observables to QCD has been achieved by means of effective theories. At low energies Chiral Perturbation Theory (ChPT) is such an effective theory[2]. In addition, the implementation of the $1/N_c$ expansion[3, 4, 5], where N_c is the number of colors in QCD, brings in constraints on the effective couplings entering in ChPT [6] increasing in this way the predictive power of the effective theory. Although this effective theory has validity within a limited low energy domain, it represents QCD faithfully, and therefore its predictions are genuine QCD predictions.

Beyond the low energy domain the availability of effective theories has been limited. In particular, in the resonance domain (excited baryons) there has not been a well established model independent analysis scheme. In order to formulate an effective theory, it is necessary to identify the small expansion parameters available. In the resonance domain, besides the light quark masses, one such expansion parameter is provided by $1/N_c$. The expansion in $1/N_c$ for excited baryons has been proposed and applied in several works [7, 8, 9, 10, 11, 12, 13] with encouraging success.

The importance of having a model independent approach to the physics of excited baryons should be emphasized. On one hand it is still mysterious how and why constituent quark models are a good qualitative, and at times also quantitative, description of that sector. It is not quite clear either what the specific deficiencies of the different versions of the quark model are. On the other hand, there is currently important experimental improvement being achieved in particular thanks to the N^* program at Jefferson Lab [14], where the study of non-strange resonances is being carried out with unprecedented precision, and also there is important progress towards studying the baryon spectrum by means of lattice QCD simulations [15]. This promising developments have rekindled the theoretical interest in

excited baryons [16]. Sorting out and understanding the physics emerging from the old and new experimental data as well as from the lattice would be greatly optimized if an effective theory is available.

Based on general arguments, it is believed that an expansion in $1/N_c$ as first proposed by 't Hooft [17] should hold in QCD in all regimes. The application of the $1/N_c$ expansion to baryons starting with the pioneering work of Witten [18] has served to understand numerous issues. In the past several years the $1/N_c$ expansion in the ground state baryons has been extensively investigated as summarized in several reviews [3, 4, 5].

Although baryons in the large N_c limit belong to increasingly large representations of the spin and flavor groups, thus giving the appearance that they are increasingly complicated, there is instead a great degree of simplification emerging in that limit. In the ground state sector it was discovered [19, 20] that the requirement that unitarity be fulfilled in π - N elastic scattering in the large N_c limit requires a dynamical spin-flavor symmetry that leads to the mentioned simplification. The reason unitarity gives rise to this non-trivial symmetry constraint is that the pion-nucleon coupling scales as $\sqrt{N_c}$, leading naively to a scattering amplitude of order N_c , which for large N_c would imply a violation of unitarity. The dynamical symmetry is a contracted $SU(2F)$ spin-flavor symmetry [19, 20], where F is the number of flavors. Up to corrections of order $1/N_c$ it is possible to replace the contracted symmetry group by the ordinary $SU(2F)$ group [21]. The possibility of building an effective theory based on this dynamical symmetry rests on the fact that for ground state baryons the symmetry is broken at order $1/N_c$. It is therefore possible to implement the $1/N_c$ expansion around the spin-flavor symmetry limit. Once the spin-flavor multiplet has been identified, which in the case of ground state baryons is the totally symmetric $SU(2F)$ representation with N_c fundamental indexes, the $1/N_c$ expansion of different observables can be carried out in terms of an expansion in composite operators [20, 22, 23, 25, 26] which are sorted according to their order in $1/N_c$. For different static observables, such as masses, magnetic moments, axial couplings, etc., using the Wigner-Eckart theorem a basis of operators can be built in terms of products of the generators of the spin-flavor group. This method was applied in the sixties [27], and when combined with the $1/N_c$ expansion it has lead to very successful analyses of the ground state baryon masses [20, 22, 23, 24, 25, 26], magnetic moments [24, 28], quadrupole moments [29], and axial couplings [28].

While the $1/N_c$ expansion is implemented rigorously along those lines for the ground

state baryons, for excited baryons there is one difficulty of principle. This has to do with the observation [8], to be made more explicit later, that in general spin-flavor symmetry is not exact for excited baryons even in the $N_c \rightarrow \infty$ limit. The breaking of spin-flavor symmetry at zeroth order in $1/N_c$ is identified in the constituent quark picture by the coupling of the orbital angular momentum. Such a breaking can give rise to spin-flavor configuration mixing, *i.e.* mixing of different spin-flavor representations, at zeroth order. As shown elsewhere [30], configuration mixing is also driven by the coupling of orbital angular momentum. This would suggest that spin-flavor symmetry cannot be used to formulate the $1/N_c$ expansion. However, it is well established from phenomenology that the orbital angular momentum couples very weakly. This is shown in analyses in the quark model [31, 32] as well as analyses in the $1/N_c$ expansion along the lines followed in this work. The zeroth order spin-flavor symmetry breaking turns out to be in the real world with $N_c = 3$ similar in magnitude to that of order $1/N_c$ breaking effects. This is illustrated by comparing the spin-orbit splitting of 115 MeV between the negative parity singlet Λ s, namely the $\Lambda(1405)$ and the $\Lambda(1520)$, to the hyperfine splittings that are of order $1/N_c$ and about 150 MeV in the corresponding negative parity states. This suggests that configuration mixing will as well be small. With this latter assumption, it is appropriate in practice to neglect configuration mixing in studying the spectrum to a level of precision of order $1/N_c$ when $N_c = 3$. In the present analysis of the negative parity baryons that belong primarily to the **70**-plet of $SU(6)$, a systematic error of order $\delta_{\text{mix}}^2/\Delta M$ where δ_{mix} is the mixing component of the mass Hamiltonian and ΔM is the splitting between the **70**-plet and the **56**-plet with which it mixes. Unfortunately no solid information exists about negative parity baryons that could be assigned primarily to a **56**-plet, and therefore no convincing estimate can at present be made about that systematic error. Thus, with the hypothesis that configuration mixing is small, the implementation of the $1/N_c$ expansion for excited baryons masses by working within a spin-flavor multiplet can be carried out along similar lines as in the ground state baryons as it was shown in [8, 10, 11].

It should be noticed that excited baryons are not narrow in the large N_c limit [7, 9, 33] as the coupling of pions and kaons mediating the transitions to ground state baryons are of order N_c^0 . For this reason, the possibility that excited baryons are built as meson-baryon resonances in large N_c is quite open. Indeed, such a possibility is well illustrated in the Skyrme model where excited baryons are built as resonances in meson-baryon scattering

[34]. Resonance models are being currently studied by various groups [35]. At this point it is important to emphasize that the $1/N_c$ analysis does not imply a specific picture of the excited baryons, as it relies entirely on group theoretical arguments and in ordering effective operators in powers of $1/N_c$, and will in particular include the possibility that excited baryons are to a large extent resonances.

In the early study of negative parity baryon masses the non-strange baryons were considered, where the expansion was carried out up to order $1/N_c^2$ [10]. Later the extension to states with non-vanishing strangeness was made in Ref. [11] where the $SU(6)$ **70**-plet masses were analyzed to order $1/N_c$ and to order ϵ where this latter parameter is a measure of the magnitude of $SU(3)$ breaking by the strange quark mass. In this work that analysis is presented in detail.

This paper is organized as follows: section II presents the construction of the negative parity baryon states, section III gives the basis of operators and their matrix elements in the **70**-plet states, section IV presents the fit to the known **70**-plet masses and mixings together with a discussion of the results of the fit, and finally section V gives a general discussion and conclusions.

II. NEGATIVE PARITY BARYON STATES

The excited baryon states that correspond in the constituent quark model to the first radial and orbital excitations fit quite well into respectively a positive parity **56**⁺ and a negative parity **70**[−] irreducible representation of the spin-flavor group $SU(6)$. Although in both cases not all the states have been experimentally determined, it seems that with those that are well established, namely those assigned a status of at least three stars by the Particle Data Group [36], it is safe to establish that observation. This conclusion is reinforced by the success of the analyses of the masses in both groups of states [10, 11, 33].

In a constituent quark picture and in the large N_c limit, the lowest baryonic excitations consist of a core of $N_c - 1$ quarks in the ground state of the Hartree effective potential, the core being therefore in the totally symmetric spin-flavor representation, and an excited quark, which for the negative parity baryons discussed in this work is in an $\ell = 1$ state [7, 8]. The states therefore fill the $(3, \mathbf{70})$ multiplet of the group $O(3) \otimes SU(6)$. Phenomenology strongly indicates that even when the current quark masses are small and the constituent

quark picture is most likely not accurate, the states can still be identified as belonging to the $(3, \mathbf{70})$. Since for the group theoretical aspects of the analysis in this work the use of the constituent quark picture can be made with no loss of generality, throughout the constituent quark terminology will be often used.

In order to explicitly build the states, it is first convenient to give a brief review of the $SU(6)$ group. It has thirty five generators, namely $\{S_i, T_a, G_{ia}\}$, with $i = 1, 2, 3$ and $a = 1, \dots, 8$, where the first three are the generators of the spin $SU(2)$, the second eight are the generators of flavor $SU(3)$, and the last twenty four can be identified as an octet of axial-vector currents in the limit of zero momentum transfer. The algebra of $SU(6)$ has the following commutation relations that fix the normalizations of the generators:

$$\begin{aligned}
[S_i, S_j] &= i\epsilon_{ijk}S_k \\
[T_a, T_b] &= if_{abc}T_c \\
[G_{ia}, G_{jb}] &= \frac{i}{4}\delta_{ij}f_{abc}T_c + \frac{i}{2}\epsilon_{ijk}(\frac{1}{3}\delta_{ab}S_k + d_{abc}G_{kc}) \\
[S_i, G_{ja}] &= i\epsilon_{ijk}G_{ka} \\
[T_a, G_{ib}] &= if_{abc}G_{ic},
\end{aligned} \tag{1}$$

where d_{abc} and f_{abc} are the usual $SU(3)$ symmetric and antisymmetric tensors, respectively. In the non-relativistic quark picture these generators can be expressed in terms of the quark fields:

$$S_i = q^\dagger \frac{\sigma_i}{2} q, \quad T_a = q^\dagger \frac{\lambda_a}{2} q, \quad G_{ia} = q^\dagger \frac{\sigma_i \lambda_a}{4} q, \tag{2}$$

where the Gell-Mann matrices are normalized as $\text{Tr} \lambda_a \lambda_b = 2\delta_{ab}$.

The states in the totally symmetric irreducible representation S are given by a Young tableau that consists of a single row of N_c boxes, and the mixed symmetric irreducible representation MS relevant to this work consists of a row with $N_c - 1$ boxes and a second row with one box.

In order to build the states belonging to the mixed symmetric $SU(6)$ irreducible representation it is convenient to start by considering the states

$$\begin{aligned}
|S S_z; (p, q), Y, I I_z; S^c > &= \sum \left(\begin{array}{c|c} S^c & \frac{1}{2} \\ \hline S_z & s_z \end{array} \middle| S \right) \left(\begin{array}{cc|c} (p^c, q^c) & (1, 0) & (p, q) \\ \hline (Y^c, I^c I_z^c) & (y, \frac{1}{2} i_z) & (Y, I I_z) \end{array} \right) \\
&\times |S^c S_z^c; (p^c, q^c), Y^c, I^c I_z^c| \frac{1}{2} s_z; (1, 0), y, \frac{1}{2} i_z \rangle,
\end{aligned} \tag{3}$$

where S is the the total spin of the baryon associated with the spin group $SU(2)$, S^c is the core spin (the core is in the S representation of $SU(6)$), Y and I are the hypercharge and isospin respectively, and (p, q) indicates the $SU(3)$ irreducible representation. For $SU(3)$ a Young tableau denoted by the pair (p, q) consists of $p + q$ boxes in the first row and q boxes in the second. From the decomposition of the S representation of $SU(6)$ as a sum of direct products of irreducible representations of $SU(2) \otimes SU(3)$ it results that $p^c + 2q^c = N_c - 1$ and $p^c = 2S^c$. This latter relation is a consequence of the fact that for the S representation the two factors in the direct products involved in the decomposition have the same Young tableau (p, q) . The rule then results from the fact that $p = 2S$ in $SU(2)$. The $p^c = 2S^c$ relation is a generalization of the so called $I = J$ rule for two flavors. The $SU(2)$ Clebsch-Gordan coefficients are defined with the standard Condon-Shortley phase convention (see e.g. Ref.[37]) while for the extra phase conventions needed to completely specify the $SU(3)$ Clebsch-Gordan coefficients the conventions in Ref. [38, 39] are followed.

Not all the states displayed in Eqn. (3) are in the MS irreducible representation of $SU(6)$. While states with $p \neq 2S$ belong automatically in the MS representation, those with $p = 2S$ are a linear combination of states in the S and MS representations. The corresponding S and MS states are easily obtained by considering in each representation the quadratic Casimir invariant of $SU(6)$, namely $C_{SU(6)}^{(2)} = 2 G_{ia} G_{ia} + \frac{1}{2} C_{SU(3)}^{(2)} + \frac{1}{3} C_{SU(2)}^{(2)}$. For the S representation $C_{SU(6)}^{(2)} = 5N_c(N_c + 6)/12$, and for the MS representation $C_{SU(6)}^{(2)} = N_c(5N_c + 18)/12$. Making use of these relations the $p = 2S$ states in the MS representation turn out to be given by

$$\begin{aligned} |S S_z; (p = 2S, q), Y, I I_z \rangle_{\text{MS}} &= \sqrt{\frac{S(N_c + 2(S + 1))}{N_c(2S + 1)}} |S S_z; (p, q), Y, I I_z; S^c = S + \frac{1}{2} \rangle \\ &- \sqrt{\frac{(S + 1)(N_c - 2S)}{N_c(2S + 1)}} |S S_z; (p, q), Y, I I_z; S^c = S - \frac{1}{2} \rangle. \end{aligned} \quad (4)$$

The states belonging to the $(3, \mathbf{70})$ of $O(3) \otimes SU(6)$ are now expressed by including the orbital angular momentum:

$$|J J_z; S; (p, q), Y, I I_z \rangle_{\text{MS}} = \sum \left(\begin{array}{cc} S & \ell \\ S_z & m \end{array} \middle| \begin{array}{c} J \\ J_z \end{array} \right) |S S_z; (p, q), Y, I I_z \rangle_{\text{MS}} | \ell m \rangle, \quad (5)$$

where J is the total angular momentum of the baryon. For $N_c = 3$ the negative parity states span the $(3, \mathbf{70})$ irreducible representation. Expressing them in the obvious notation $^{2S+1}d_J$, they are as follows: five $SU(3)$ octets ($^2 8_{\frac{1}{2}}, ^2 8_{\frac{3}{2}}, ^4 8_{\frac{1}{2}}, ^4 8_{\frac{3}{2}}, ^4 8_{\frac{5}{2}}$), two decouplets ($^2 10_{\frac{1}{2}}, ^2 10_{\frac{3}{2}}$),

and two singlets ($^2 1_{\frac{1}{2}}, ^2 1_{\frac{3}{2}}$). The corresponding states in these multiplets are shown in the first column of Table VI.

The octets ($p = 1$) of spin 1/2 satisfy $p = 2S$ and therefore involve a linear combination of core states as specified in Eqn. (4). All other states have $p \neq 2S$ and have therefore core states with well defined spin: the spin 3/2 octets as well as the decouplets ($p = 3$) have $S^c = 1$, and the two singlet Λ states ($p = 0$) have $S^c = 0$.

In the limit of exact $SU(3)$ symmetry there are two possible mixings induced by interactions that break spin-flavor symmetry. These mixings are between the pairs of states that are in the two octets with same J . The mixing angles are defined according to:

$$\begin{pmatrix} 8_J \\ 8'_J \end{pmatrix} = \begin{pmatrix} \cos \theta_{2J} & \sin \theta_{2J} \\ -\sin \theta_{2J} & \cos \theta_{2J} \end{pmatrix} \begin{pmatrix} ^2 8_J \\ ^4 8_J \end{pmatrix}, \quad (6)$$

where $J = \frac{1}{2}$ and $\frac{3}{2}$, $8_J^{(')}$ are mass eigenstates, and the mixing angles are constrained to be in the interval $[0, \pi)$. At this point it is easy to check that in the $SU(3)$ limit there are nine masses and two mixing angles, i.e., eleven observables.

III. MASS OPERATORS

In the subspace of the MS states, the mass operator can be expressed in terms of a linear combination of composite operators sorted according to their order in $1/N_c$. A basis of such composite operators can be constructed using the $O(3) \otimes SU(6)$ generators, distinguishing two sets according to whether the generator acts on the core or on the excited quark. Generators acting on the core will be denoted by $\{S_i^c, T_a^c, G_{ia}^c\}$ and those acting on the excited quark by $\{s_i, t_a, g_{ia}\}$. Operators can be classified according to their n -body character. Thus, operators containing a product of n $SU(6)$ generators acting only on the core, and operators containing a product of $n-1$ $SU(6)$ generators acting on the core and at least one generator of $O(3) \otimes SU(6)$ acting on the excited quark, are said to be n -body operators. The $1/N_c$ counting can then be determined by the following two criteria:

i) An n -body operator requires that at least $(n-1)$ gluons be exchanged between the n quarks, and thus, the coefficient that multiplies the operator in the effective theory is proportional to $1/N_c^{(n-1)}$. Henceforth, this factor will be absorbed in the definition of the operator.

ii) In the large N_c limit the $SU(6)$ generators G_{ia}^c with $a = 1, 2, 3$, and T_8^c have matrix elements of order N_c between states with spin and strangeness of order N_c^0 , and they are therefore called coherent generators. The presence of coherent factors in a composite operator leads to an enhancement of its matrix elements between such states given by a power of N_c . In order to determine that power it is necessary to first reduce the products of generators by means of the commutation relations.

At this point it should be noted that there is an ambiguity in the identification of the physical states for $N_c = 3$ with the states in the large N_c spin-flavor multiplet. As in previous works, we resolve this ambiguity by identifying the physical states with those of spin and strangeness of order N_c^0 so that (ii) can be applied.

In the construction of composite operators identities for certain products of generators valid in a given irreducible representation of $SU(6)$ should be used. These identities or reduction rules have been given in Ref. [25] for the case of the S representation of $SU(6)$. Those relevant to the present work are applied to products of generators acting on the core and are the following ones:

$$\begin{aligned}
12\{G_{ia}^c, G_{ia}^c\} &= 5(N_c - 1)(N_c + 5) - 4S_c^2 - 3\{T_a^c, T_a^c\} \\
\{T_a^c, T_a^c\} &= 2S_c^2 + \frac{(N_c - 1)(N_c + 5)}{6} \\
\{G_{ia}^c, G_{ja}^c\} |_{J=2} &= \frac{1}{3}\{S_i^c, S_j^c\} |_{J=2} \\
d_{abe}\{G_{ia}^c, G_{jb}^c\} |_{J=2} &= \frac{1}{3}\{S_i^c, G_{je}^c\} |_{J=2} .
\end{aligned} \tag{7}$$

In particular, the first reduction rule results from the quadratic Casimir, the second results from the $p^c = 2S^c$ rule mentioned earlier.

The mass operators in the basis must be rotationally invariant, parity and time reversal even and isospin symmetric. At order N_c the basis of $SU(3)$ singlet operators consist of one operator, namely the identity operator that essentially counts the number of valence quarks in the baryon and, therefore, preserves spin-flavor symmetry. At order N_c^0 only operators involving factors of the orbital angular momentum appear. It is not difficult to understand why there are operators of order N_c^0 : by looking at the constituent quark picture, the excited quark moves in the effective potential of order N_c^0 generated by the core giving rise to a spin-orbit interaction (e.g. the $\ell \cdot s$ operator) with a strength of order N_c^0 . The matrix elements of the excited quark spin in the MS representation are of order

N_c^0 , and so the spin-orbit interaction is of that order as well [40]. There are three linearly independent operators of order N_c^0 , the operators O_2 , O_3 and O_4 listed in Table I. The 1-body operator O_2 is the ordinary spin-orbit operator, while the remaining operators are 2-body and involve factors carrying flavor. The dynamics giving rise to composite operators involving flavor exchange is not understood but it is likely that long distance effects due to the pion and kaon clouds give an important part of the strength to these operators. In particular pion-exchange quark models [41, 42] lead naturally to important flavor exchange contributions. It is a straightforward exercise to check that any other operators of order N_c^0 are linearly dependent with the identity and the three operators of order N_c^0 up to terms of order $1/N_c$. There are eleven operators of order $1/N_c$ in the basis. Since for $N_c = 3$ there are eleven $SU(3)$ singlet mass observables as indicated earlier, and four basis operators have already been identified at orders N_c and N_c^0 , it results that for $N_c = 3$ only seven operators of order $1/N_c$ are independent. We consider those listed in Table I. Among them is the very important hyperfine operator O_6 , known to play a crucial role in baryon spectroscopy. Note that there are also three 3-body operators as well. At this point it is opportune to give a further understanding of the singlet operators based on group theory. For $N_c = 3$ the $SU(3)$ singlets are those contained in the $SU(2) \otimes SU(3)$ decomposition of the product $\overline{\mathbf{70}} \otimes \mathbf{70}$, namely:

$$\begin{aligned} \overline{\mathbf{70}} \otimes \mathbf{70} = & 4(\mathbf{1}, \mathbf{1}) \oplus 5(\mathbf{3}, \mathbf{1}) \oplus 2(\mathbf{5}, \mathbf{1}) \oplus (\mathbf{7}, \mathbf{1}) \\ & \oplus 7(\mathbf{1}, \mathbf{8}) \oplus 11(\mathbf{3}, \mathbf{8}) \oplus 6(\mathbf{5}, \mathbf{8}) \oplus 2(\mathbf{7}, \mathbf{8}) \oplus \dots \end{aligned} \quad (8)$$

which shows that there are four singlet operators with $\ell = 0$ (O_1 , O_6 , O_7 , O_{10}), five with $\ell = 1$ (O_2 , O_4 , O_5 , O_9 , O_{11}), two with $\ell = 2$ (O_3 , O_8) and one operator with $\ell = 3$ that obviously cannot contribute to matrix elements between states with $\ell = 1$.

The breaking of $SU(3)$ symmetry is driven by the mass difference $m_s - \hat{m}$, where \hat{m} is the average of the u- and d-quark masses. A measure of the breaking is given by the ratio $\epsilon = (M_K^2 - M_\pi^2)/\Lambda^2$ where Λ is a light hadronic scale, for instance a vector meson mass. Here $SU(3)$ breaking will be included to order ϵ . Note that for $N_c = 3$, ϵ and $1/N_c$ are of similar size, and therefore corrections of order ϵ/N_c are neglected. At order ϵ the effective operators are obviously octets. Explicit construction gives four basis operators, two 1-body and two 2-body operators. Listed in Table I are improved operators obtained by combining the octet pieces with singlet operators in such a way that the resulting operators, $\bar{B}_{1,\dots,4}$,

have vanishing matrix elements between non-strange baryons. Note that the operator \bar{B}_2 consists of two pieces of order N_c , namely T_8^c and the operator O_1 . However, the order N_c pieces cancel and \bar{B}_2 is actually of order N_c^0 . It should be mentioned that, for reasons explained below, several singlet and octet operators differ from those in [10] and [11] by a scaling factor.

The calculation of the matrix elements of these operators in the basis described in Section II is a rather lengthy task. For this purpose the Wigner-Eckart theorem has been repeatedly used to express any given matrix element in terms of combinations of $SU(2)$ and $SU(3)$ Clebsch-Gordan coefficients and the reduced matrix elements of the elementary operators acting on the core and excited quark, $\{S_i^c, T_a^c, G_{ia}^c\}$ and $\{s_i, t_a, g_{ia}\}$, respectively. As usual such reduced matrix elements were expressed in terms of Clebsch-Gordan coefficients and some particular matrix elements which are simple to evaluate explicitly. Fortunately, for the cases of interest there exist analytic formulas for all the $SU(2)$ coefficients[37] and $SU(3)$ isoscalar factors [38, 39] involved in the calculations. Consequently, it has been possible to derive explicit expressions, in terms of N_c , for all the matrix elements of the singlet and octet operators included in the present analysis. Such expressions are given in Tables II-V. It should be mentioned that all these formulas have been independently checked by means of a numerical method based on a standard $SU(3)$ Wigner-coefficients package[43]. The matrix elements of the singlet operators in the sector of non-singlet states can also be checked by comparing with those obtained in the $SU(4)$ analysis in [10]. Indeed, with the correspondences $O_i \rightarrow \tilde{O}_i$ $i = 1, 2, 5 - 9$, and $O_3 \rightarrow \tilde{O}_3 + \frac{1}{8}\tilde{O}_8$, $O_4 \rightarrow \tilde{O}_4 + \frac{N_c}{3(N_c+1)}\tilde{O}_5$, where the \tilde{O}_i are the operators in $SU(4)$ differing with those in [10] by some of the rescaling factors that were mentioned earlier, it is straightforward to cross-check the results.

It is also possible to check that certain combinations of operators are demoted to become of higher order in $1/N_c$ in the sector of non-singlet baryons. As it was observed in [10] the combination of zeroth order operators $O_2 + O_4$ is of order $1/N_c$ and the combination of zeroth and first order operators $O_2 + O_4 + \frac{2}{3}O_5 + \frac{8}{3}O_9$ is of order $1/N_c^2$ in that sector.

IV. FITS AND DISCUSSIONS

In terms of the basis of operators introduced in the previous section the **70**-plet mass operator up to order $1/N_c$ and order ϵN_c^0 has the most general form:

$$M_{\mathbf{70}} = \sum_{i=1}^{11} c_i O_i + \sum_{i=1}^4 d_i \bar{B}_i \quad , \quad (9)$$

where c_i and d_i are the unknown real coefficients to be determined by fitting to the known masses and mixing angles. These coefficients encode the non-perturbative QCD dynamics that cannot be constrained by symmetries. Calculating these coefficients would be equivalent to solving QCD in this baryon sector. Fortunately, the experimental data available in the case of the **70**-plet is enough to obtain them by performing a fit[11]. The inputs to the fit consist of seventeen masses of negative parity baryons which have been assigned a status of three or more stars by the Particle Data Group [36], and the two leading order mixing angles $\theta_1 = 0.61 \pm 0.09$ and $\theta_3 = 3.04 \pm 0.15$ on which there is a rather good consensus about their values as obtained from strong decays of the non-strange members of the multiplet [7, 31, 44]. Note that θ_3 is consistent with zero mod π . Thus, the fifteen coefficients are fitted to these nineteen observables. Of course, a larger number of inputs would have been desirable in order to increase the confidence level of the results. It is however expected that the chief features of the results that are found here are sufficiently well established with these inputs.

Before proceeding to the numerical analysis of the masses, it is important to establish relations among observables that must hold to the order of this analysis. Including $SU(3)$ breaking to all orders there are fifty observables, namely thirty masses and twenty mixing angles. Since there are fifteen operators in the basis being considered, there must be thirty five relations. On the other hand, if $SU(3)$ breaking is considered only to order ϵ , there are thirty five observables where twenty one of them are masses and fourteen are mixing angles. This then implies that up to order ϵ and order $1/N_c$ there are twenty relations. Among these relations there are those independent of the leading order mixing angles resulting among traces of mass matrices for states with the same quantum numbers I and J . There are thirteen such relations, which are independent of the coefficients c_i and b_i , all of them involving mass splittings. Five of them are Gell-Mann Okubo relations (one per octet), four equal spacing rules (two per decouplet), and four novel relations that involve mass splittings

across $SU(3)$ multiplets. These relations are given by:

$$\begin{aligned}
14(s_{\Lambda_{3/2}} + s_{\Lambda'_{3/2}}) + 63s_{\Lambda_{5/2}} + 36(s_{\Sigma_{1/2}} + s_{\Sigma'_{1/2}}) &= 68(s_{\Lambda_{1/2}} + s_{\Lambda'_{1/2}}) + 27s_{\Sigma_{5/2}} \\
14(s_{\Sigma_{3/2}} + s_{\Sigma'_{3/2}}) + 21s_{\Lambda_{5/2}} - 9s_{\Sigma_{5/2}} &= 18(s_{\Lambda_{1/2}} + s_{\Lambda'_{1/2}}) + 2(s_{\Sigma_{1/2}} + s_{\Sigma'_{1/2}}) \\
14s_{\Sigma''_{1/2}} + 49s_{\Lambda_{5/2}} + 23(s_{\Sigma_{1/2}} + s_{\Sigma'_{1/2}}) &= 45(s_{\Lambda_{1/2}} + s_{\Lambda'_{1/2}}) + 19s_{\Sigma_{5/2}} \\
14s_{\Sigma''_{3/2}} + 28s_{\Lambda_{5/2}} + 11(s_{\Sigma_{1/2}} + s_{\Sigma'_{1/2}}) &= 27(s_{\Lambda_{1/2}} + s_{\Lambda'_{1/2}}) + 10s_{\Sigma_{5/2}}, \tag{10}
\end{aligned}$$

where s_{B_i} is the mass splitting between the state B_i and the non-strange states in the $SU(3)$ multiplet to which it belongs. For the purpose of identifying the states within an $SU(3)$ multiplet the order ϵ mixing is disregarded. It is important to stress here that these relations are independent of the leading order as well as the order ϵ mixings.

As already mentioned, several of the singlet and octet operators defined in this article differ from those in [10] and [11] by a scaling factor. The reason for this is that here the operators have been defined in such a way that their matrix elements are of the same order in $1/N_c$ at which the operator contributes. With this, the natural size of the coefficients of singlet operators is about 500 MeV as the analysis below shows, and that of the coefficients of $SU(3)$ breaking is about $\epsilon \times 500 \text{ MeV} \sim 150 - 200 \text{ MeV}$.

The fit has been performed by treating the singlet pieces of the mass operator exactly and the $SU(3)$ breaking to first order of perturbation theory in ϵ . This approach is justified in practice by the fact that the hyperfine interaction turns out to be the dominant spin-flavor breaking piece. The results of the fit are given in Table I, where the natural size of the coefficients associated with the singlet operators is seen to be set by the coefficient of O_1 . In fitting the data the experimental errors given by the Particle Data Group [36] are taken whenever they are larger than the expected magnitude of higher order corrections in the analysis. These corrections are of order ϵ^2 or ϵ/N_c , and their magnitude is taken to be about 15 MeV. Although this is not crucial for the outcome of the fit, the resulting χ^2 is more realistic. For instance, the singlet Λ masses are known experimentally within 5 MeV, and taking this error, which in magnitude would correspond to a higher order of precision in the expansion in ϵ and $1/N_c$, would be unrealistic.

Table VI displays the empirical masses, and the masses and compositions of states resulting from the fit, whose χ^2 per degree of freedom turns out to be 1.29. A clear display of the results is shown in Figs. 1-3, where the results are plotted along with the experimental values as well as the best fit provided by the Isgur-Karl model[31].

In order to better understand the outcome of the fit, it is convenient to first emphasize the hierarchy that emerges from the analysis. The contributions by the singlet operators to different mass matrix elements can be seen by considering the contributions to the non-strange baryons and the singlet Λ 's. Table VII explicitly shows such contributions. As it was already found in the analysis of the non-strange baryons [10], the singlet operators of order N_c^0 give contributions to the masses that are much smaller than the natural size expected at this order. Indeed, they turn out to be of similar or even smaller magnitude than the natural size expected at order $1/N_c$. Their importance however shows in the leading order mixings driven by the matrix elements shown in the first two rows of Table VII. Among the operators of order $1/N_c$, the dominant operator is the hyperfine operator O_6 that gives the chief spin-flavor breaking, with all other operators giving contributions that are suppressed with respect to the natural size. This observed hierarchy that goes beyond the simple ordering in powers of $1/N_c$ reflects the dynamics of QCD, and it is interesting to note that in its general aspects it agrees with the hierarchy that results in constituent quark models.

A more detailed description of the role of the different singlet operators is the following:

- The hyperfine operator O_6 gives the gross spin-flavor breaking features in the **70**-plet. This operator does not affect the singlet Λ states which involve core states with $S^c = 0$ only, while it increases the masses of the rest of the states according to the $S^c = 1$ content of the cores. The typical mass shifts produced by this operator are 160 MeV, which is the natural size expected from $1/N_c$ counting alone. In particular, this makes clear why the singlet Λ 's are the lightest states in the **70**-plet, as it is well known from the early works in the constituent quark model based on QCD [45]. In the large N_c limit the hyperfine interaction in the core should be the same as in the ground state baryons. The $\Delta - N$ splitting gives for the ground state baryons a strength of 100 MeV for the hyperfine interaction defined by $\sum_{i \neq j}^{N_c} s_i \cdot s_j$, while the corresponding strength implied by the result obtained for the coefficient c_6 is equal to 150 MeV. This disagreement is a manifestation of higher order corrections in $1/N_c$ and is in line with the expected magnitude for $N_c = 3$. It should be mentioned here that, as it occurs in the ground state baryons, the hyperfine operator has actually coherent matrix elements between states with spin of order N_c , for which the splittings between states are of zeroth order.

- The spin-orbit operator O_2 has the peculiarity of being one of the two operators that affect the singlet Λ masses (note that improved $SU(3)$ breaking operators also do, but only through their singlet pieces proportional to the unity and spin-orbit operators, since the octet piece has vanishing matrix elements for these states). The splitting between the singlet Λ 's is therefore a direct measure of the spin-orbit interaction. The fact that the splitting is only 113 MeV indicates the weakness of the spin-orbit effect. This is perhaps the best indication that the formal problem originating in the fact that spin-flavor symmetry is broken at order N_c^0 is rather harmless in practice. Understanding the smallness of the spin-orbit interaction from QCD is an important open dynamical problem. In the constituent quark model picture a possible explanation has been given in terms of the approximate cancellation between the spin-orbit pieces due to the Lorentz four-vector and scalar components of the effective potential [46, 47]. The sign of the spin-orbit splitting between the Λ singlets obtained here indicates that within that picture the Lorentz four-vector effective potential should give the larger contribution in the case of the **70**-plet.

Note that the coefficient c_2 has a much smaller error than the combination $c_2 + c_4$ in the non-strange sector analysis [10] with which it ought to be compared. This is a consequence of the very important role of the singlet Λ states that help pin down the contribution of O_2 much more precisely.

One interesting observation can be made concerning the splittings between spin-orbit partners in the **56**-plet. The positive parity **56**-plet with $\ell = 2$ containing the spin-orbit partner N^* states $N_{3/2}(1720)$ and $N_{5/2}(1680)$, and the Δ^* states $\Delta_{1/2}(1910)$, $\Delta_{3/2}(1920)$, $\Delta_{5/2}(1905)$ and $\Delta_{7/2}(1950)$ [36] shows very small splittings. Indeed, these splittings are suppressed by a factor one third or smaller with respect to the spin-orbit splitting between the singlet Λ states in the **70**-plet. Since the coupling of orbital angular momentum within the **56**-plet is of order $1/N_c$, it is possible that this suppression is just a manifestation of the $1/N_c$ expansion.

- The remaining two operators of order N_c^0 involve flavor exchange and give also contributions that are rather small, but important in the understanding of two issues.
 - i) The first issue is the so called spin-orbit puzzle in the quark model that can be summarized by the incompatibility between such splittings in the sector of non-strange

baryons and in the singlet Λ 's. The operator O_4 gives contributions that compensate those of O_2 to the splitting between the spin-orbit partner states $\Delta_{1/2}$ and $\Delta_{3/2}$, where the manifestation of the spin-orbit puzzle was most dramatic. The operators O_3 and O_5 do also give some relevant contributions to such splittings for other states, but they are smaller than those of O_4 . A conclusion that can be drawn here is that the spin-orbit puzzle in quark models is resolved by the flavor-exchange effective interactions not included in that model and that appear naturally in the $1/N_c$ analysis.

ii) The second issue is the leading order mixings that are due to the off diagonal matrix elements in the two octet mass matrices. In particular, the mixing angle θ_1 , which is the only significant one of the two leading order angles, is almost entirely determined by the operator O_3 , as the partial contributions displayed in Table VII show. The angle θ_3 receives contributions from several operators ($O_{2,\dots,5,9,11}$) that are of similar magnitude and tend to cancel. The first two rows in Table VII clearly show that the contributions to mixings by the order $1/N_c$ operators are small and even more tend to cancel each other to a substantial extent.

- The hyperfine operator O_7 gives very small contributions. This operator involves the spin-spin interactions between the excited quark and the core, which according to the constituent quark picture is suppressed by the centrifugal barrier. As shown in Table VII, the current analysis shows that this operator gives splittings much smaller than those by O_6 and of the order of 25 MeV, in qualitative agreement with the quark picture.
- The operator O_5 and the three-body operators give contributions whose magnitude is in the few tens of MeV, i.e. much smaller than the natural size of $1/N_c$ contributions, and have no clearly definite effect with which they could be associated. They do however contribute to the ultimate quality of the fit. Finally, the operator O_8 is clearly irrelevant.
- Table VIII shows the results of partial fits to the masses of the non-strange and the singlet Λ baryons listed in the first column of Table VII. The contribution of the strange quark to the Λ masses has been taken to be 135 MeV as it results from the average empirical splitting per unit of strangeness. The progress in the quality of

the fit can be observed as operators are sequentially included. Comparing the fitted coefficients with the values obtained from using the full basis $O_{1,\dots,11}$, *i.e.* those listed in the last line of Table VIII (note that these values can be exactly obtained since there are eleven observables) it can be noticed that the values obtained in successive partial fits always remain close to them.

Concerning $SU(3)$ breaking, only three out of the four $SU(3)$ breaking operators are significant giving natural size contributions. \bar{B}_3 is weak and can to some extent be disregarded. The dominance of the O_6 operator may indicate that associated octet operators such as $\frac{1}{N_c} S_i^c G_{i8}^c$, which is of order ϵ/N_c , would be important, even when it appears at higher order than the ones considered in this paper. Although this may be so, the available data for splittings does not allow to pin down the relevance of such an operator. The fact is that with the four improved leading order operators already included the fit is very good, and the inclusion of such operator does not lead to a significant improvement. More data on $SU(3)$ splittings would be required to clarify this issue. The inclusion of such an operator would spoil the splitting relations of Eq. (10). The main observations on $SU(3)$ breaking are the following:

- Only one relation can be tested with available data, namely the Gell-Mann Okubo relation for the $J = 3/2$ octet that is predominantly $S = 1/2$. Determining the masses of the $\Xi'_{1/2}$ and $\Xi_{5/2}$ would complete two more octets and test the corresponding relations. A further octet can be completed that has one two-star state, the $\Sigma_{1/2}(1620)$ state, by finding the corresponding Ξ state predicted by the analysis to have a mass of 1779 MeV.
- The test of equal spacing relations is not possible. Only the Δ states in the two decouplets are known. It is clearly very important at some point in the future to have further decouplet states experimentally pinned down for that purpose. The results obtained here indicate that the splittings in both decouplets are similar and in the range 125–135 MeV, which is the typical splitting produced by one unit of strangeness.
- The Gell-Mann Okubo and equal spacing relations are violated at order ϵ^2 , while the four relations Eq. (10) are violated at order ϵ/N_c and ϵ^2 . While in the ground state baryons the Gell-Mann Okubo and equal spacing relations are violated at order ϵ^2/N_c

[26], the order ϵ^2 violation of the relations in excited baryons results primarily from the $SU(3)$ breaking mixings between Σ s and between Λ s belonging to different multiplets.

- The four new splitting relation of Eq.(10) cannot be tested at this point because the masses of $\Lambda'_{3/2}$, $\Sigma'_{3/2}$, $\Sigma''_{1/2}$, and $\Sigma'_{1/2}$ that enter respectively in the four relations need to be known. After replacing the known experimental values, each of the splitting relations gives a prediction, namely: $s_{\Lambda'_{3/2}} = 149$ MeV, $s_{\Sigma'_{3/2}} = 55$ MeV, $s_{\Sigma''_{1/2}} = 154$ MeV, and $s_{\Sigma'_{1/2}} = 134$ MeV. If the operator \bar{B}_3 is ignored, five relations result that were given in [11]. In that case one relation could be tested, namely the relation $9(s_{\Sigma_{1/2}} + s_{\Sigma'_{1/2}}) + 21s_{\Lambda_{5/2}} = 17(s_{\Lambda_{1/2}} + s_{\Lambda'_{1/2}}) + 5s_{\Sigma_{5/2}}$; by including the two-star state $\Sigma_{1/2}(1620)$ as input that relation is satisfied to a few percent.

V. CONCLUSIONS

The $1/N_c$ expansion for excited baryons has been implemented under the assumption that there is an approximate spin-flavor symmetry in the large N_c limit. This assumption relies on the observation that zeroth order violations of this symmetry are very small in practice. Consequently, the only effects that have been left out in the analysis carried out for the $SU(6)$ **70**-plet are related to spin-flavor configuration mixing. Since the analysis shows that the zeroth order spin-flavor breaking in the **70**-plet has a magnitude smaller than the natural size of first order contributions, the scheme is phenomenologically sound.

The analysis also shows that the $1/N_c$ expansion can be consistently applied because there are no corrections that are unnaturally large. On the other hand, a hierarchy emerges in the form of effective coefficients being unnaturally small. In its gross features the picture that emerges is similar to the quark model one, but at a finer level the suppressed dynamics manifests itself in particular through flavor exchange effective interactions, largely absent in most quark models, which are important in describing two effects, namely, the zeroth order mixings and the resolution of the spin-orbit puzzle.

At the level of $SU(3)$ singlet spin-flavor symmetry breaking the level of predictivity is quite limited, the reason being that the number of observables is equal to the number of operators in the singlet basis up to order $1/N_c$. There is however predictivity at the level of $SU(3)$ breaking to order $\epsilon \times N_c^0$: besides Gell-Mann Okubo and equal spacing relations there are four new relations across different $SU(3)$ multiplets. Unfortunately, with the available

data only one Gell-Mann Okubo relation can be tested. This should be a motivation to experimentally establish a few more key states in the **70**-plet.

The present analysis provides a useful framework to sort out and understand results from lattice QCD simulations of excited baryons. The $1/N_c$ expansion allows to separate the contributions that follow from the dynamical $SU(2F)$ symmetry and its breaking, from the non-perturbative reduced matrix elements of the QCD operators. In particular, the $\Lambda(1405)$ appears naturally as the lightest state and a spin-orbit partner of the $\Lambda(1520)$. The spin-spin and spin-orbit interactions that give the gross structure of the **70**-plet are especially interesting and lattice simulations together with the $1/N_c$ analysis could help to further understand their nature.

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TABLE I: List of operators and the coefficients resulting from the best fit to the known **70**-plet masses and mixings [11].

Operator	Fitted coef. [MeV]
$O_1 = N_c \, 1$	$c_1 = 449 \pm 2$
$O_2 = l_i \, s_i$	$c_2 = 52 \pm 15$
$O_3 = \frac{3}{N_c} \, l_{ij}^{(2)} \, g_{ia} \, G_{ja}^c$	$c_3 = 116 \pm 44$
$O_4 = \frac{4}{N_c+1} \, l_i \, t_a \, G_{ia}^c$	$c_4 = 110 \pm 16$
$O_5 = \frac{1}{N_c} \, l_i \, S_i^c$	$c_5 = 74 \pm 30$
$O_6 = \frac{1}{N_c} \, S_i^c \, S_i^c$	$c_6 = 480 \pm 15$
$O_7 = \frac{1}{N_c} \, s_i \, S_i^c$	$c_7 = -159 \pm 50$
$O_8 = \frac{2}{N_c} \, l_{ij}^{(2)} \, s_i \, S_j^c$	$c_8 = 3 \pm 55$
$O_9 = \frac{3}{N_c^2} \, l_i \, g_{ja} \{S_j^c, G_{ia}^c\}$	$c_9 = 71 \pm 51$
$O_{10} = \frac{2}{N_c^2} \, t_a \{S_i^c, G_{ia}^c\}$	$c_{10} = -84 \pm 28$
$O_{11} = \frac{3}{N_c^2} \, l_i \, g_{ia} \{S_j^c, G_{ja}^c\}$	$c_{11} = -44 \pm 43$
$\bar{B}_1 = t_8 - \frac{1}{2\sqrt{3}N_c} O_1$	$d_1 = -81 \pm 36$
$\bar{B}_2 = T_8^c - \frac{N_c-1}{2\sqrt{3}N_c} O_1$	$d_2 = -194 \pm 17$
$\bar{B}_3 = \frac{10}{N_c} \, d_{8ab} \, g_{ia} \, G_{ib}^c + \frac{5(N_c^2-9)}{8\sqrt{3}N_c^2(N_c-1)} O_1 +$ $\quad + \frac{5}{2\sqrt{3}(N_c-1)} O_6 + \frac{5}{6\sqrt{3}} O_7$	$d_3 = -15 \pm 30$
$\bar{B}_4 = 3 \, l_i \, g_{i8} - \frac{\sqrt{3}}{2} O_2$	$d_4 = -27 \pm 19$

TABLE II: Matrix elements of singlet operators O_1 to O_6 in the **70**-plet. Note that although it is not explicitly indicated, within each subspace the matrix elements which are not diagonal with respect to isospin I and strangeness S vanish.

	O_1	O_2	O_3	O_4	O_5	O_6
	$N_c \mathbf{1}$	$l_i s_i$	$\frac{3}{N_c} l_{ij}^{(2)} g_{ia} G_{ja}^c$	$\frac{4}{N_c+1} l_i t_a G_{ia}^c$	$\frac{1}{N_c} l_i S_i^c$	$\frac{1}{N_c} S_i^c S_i^c$
$^2 8_{\frac{1}{2}}$	N_c	$\frac{3-2N_c}{3N_c}$	0	$\frac{2}{9} \frac{(N_c+3)(3N_c-2)}{N_c(N_c+1)}$	$-\frac{N_c+3}{3N_c^2}$	$\frac{N_c+3}{2N_c^2}$
$^4 8_{\frac{1}{2}}$	N_c	$-\frac{5}{6}$	$-\frac{5(3N_c+1)}{48N_c}$	$\frac{5(3N_c+1)}{18(N_c+1)}$	$-\frac{5}{3N_c}$	$\frac{2}{N_c}$
$^2 8_{\frac{1}{2}} - ^4 8_{\frac{1}{2}}$	0	$-\frac{1}{3\sqrt{2}} \sqrt{1+\frac{3}{N_c}}$	$-\frac{5(3N_c-2)}{24\sqrt{2}N_c} \sqrt{1+\frac{3}{N_c}}$	$-\frac{1}{9\sqrt{2}} \frac{5-3N_c}{N_c+1} \sqrt{1+\frac{3}{N_c}}$	$\frac{1}{3\sqrt{2}N_c} \sqrt{1+\frac{3}{N_c}}$	0
$^2 1_{\frac{1}{2}}$	N_c	-1	0	0	0	0
$^2 10_{\frac{1}{2}}$	N_c	$\frac{1}{3}$	0	$-\frac{(3N_c+7)}{9(N_c+1)}$	$-\frac{4}{3N_c}$	$\frac{2}{N_c}$
$^2 8_{\frac{3}{2}}$	N_c	$\frac{2N_c-3}{6N_c}$	0	$-\frac{1}{9} \frac{(N_c+3)(3N_c-2)}{N_c(N_c+1)}$	$\frac{N_c+3}{6N_c^2}$	$\frac{N_c+3}{2N_c^2}$
$^4 8_{\frac{3}{2}}$	N_c	$-\frac{1}{3}$	$\frac{3N_c+1}{12N_c}$	$\frac{3N_c+1}{9(N_c+1)}$	$-\frac{2}{3N_c}$	$\frac{2}{N_c}$
$^2 8_{\frac{3}{2}} - ^4 8_{\frac{3}{2}}$	0	$-\frac{\sqrt{5}}{6} \sqrt{1+\frac{3}{N_c}}$	$-\frac{\sqrt{5}(2-3N_c)}{48N_c} \sqrt{1+\frac{3}{N_c}}$	$-\frac{\sqrt{5}}{18} \frac{5-3N_c}{N_c+1} \sqrt{1+\frac{3}{N_c}}$	$\frac{\sqrt{5}}{6N_c} \sqrt{1+\frac{3}{N_c}}$	0
$^2 1_{\frac{3}{2}}$	N_c	$\frac{1}{2}$	0	0	0	0
$^2 10_{\frac{3}{2}}$	N_c	$-\frac{1}{6}$	0	$\frac{3N_c+7}{18(N_c+1)}$	$\frac{2}{3N_c}$	$\frac{2}{N_c}$
$^4 8_{\frac{3}{2}}$	N_c	$\frac{1}{2}$	$-\frac{3N_c+1}{48N_c}$	$-\frac{3N_c+1}{6(N_c+1)}$	$\frac{1}{N_c}$	$\frac{2}{N_c}$

TABLE III: Matrix elements of singlet operators O_7 to O_{11} in the **70**-plet. Note that although it is not explicitly indicated, within each subspace the matrix elements which are not diagonal with respect to isospin I and strangeness S vanish.

	O_7	O_8	O_9	O_{10}	O_{11}
	$\frac{1}{N_c} s_i S_i^c$	$\frac{2}{N_c} l_{ij}^{(2)} s_i S_j^c$	$\frac{3}{N_c^2} l_i g_{ja} \{S_j^c, G_{ia}^c\}$	$\frac{2}{N_c^2} t_a \{S_i^c, G_{ia}^c\}$	$\frac{3}{N_c^2} l_i g_{ia} \{S_j^c, G_{ja}^c\}$
$^2 8_{\frac{1}{2}}$	$-\frac{(N_c+3)}{4N_c^2}$	0	$\frac{(N_c+3)(7-15N_c)}{24N_c^3}$	$-\frac{(N_c+3)(3N_c+1)}{12N_c^3}$	$-\frac{(N_c+3)(3N_c+1)}{24N_c^3}$
$^4 8_{\frac{1}{2}}$	$\frac{1}{2N_c}$	$\frac{5}{3N_c}$	$\frac{5(3N_c+1)}{24N_c^2}$	$-\frac{(3N_c+1)}{3N_c^2}$	$\frac{5(3N_c+1)}{12N_c^2}$
$^2 8_{\frac{1}{2}} - ^4 8_{\frac{1}{2}}$	0	$\frac{5}{6\sqrt{2}N_c} \sqrt{1+\frac{3}{N_c}}$	$\frac{3N_c-2}{12\sqrt{2}N_c^2} \sqrt{1+\frac{3}{N_c}}$	0	$\frac{3N_c+1}{6\sqrt{2}N_c^2} \sqrt{1+\frac{3}{N_c}}$
$^2 1_{\frac{1}{2}}$	0	0	0	0	0
$^2 10_{\frac{1}{2}}$	$-\frac{1}{N_c}$	0	$\frac{(3N_c+7)}{6N_c^2}$	$\frac{(3N_c+7)}{6N_c^2}$	$\frac{(3N_c+7)}{12N_c^2}$
$^2 8_{\frac{3}{2}}$	$-\frac{(N_c+3)}{4N_c^2}$	0	$\frac{(N_c+3)(15N_c-7)}{48N_c^3}$	$-\frac{(N_c+3)(3N_c+1)}{12N_c^3}$	$\frac{(N_c+3)(3N_c+1)}{48N_c^3}$
$^4 8_{\frac{3}{2}}$	$\frac{1}{2N_c}$	$-\frac{4}{3N_c}$	$\frac{(3N_c+1)}{12N_c^2}$	$-\frac{(3N_c+1)}{3N_c^2}$	$\frac{(3N_c+1)}{6N_c^2}$
$^2 8_{\frac{3}{2}} - ^4 8_{\frac{3}{2}}$	0	$-\frac{\sqrt{5}}{12N_c} \sqrt{1+\frac{3}{N_c}}$	$\frac{\sqrt{5}(3N_c-2)}{24N_c^2} \sqrt{1+\frac{3}{N_c}}$	0	$\frac{\sqrt{5}(3N_c+1)}{12N_c^2} \sqrt{1+\frac{3}{N_c}}$
$^2 1_{\frac{3}{2}}$	0	0	0	0	0
$^2 10_{\frac{3}{2}}$	$-\frac{1}{N_c}$	0	$-\frac{(3N_c+7)}{12N_c^2}$	$\frac{(3N_c+7)}{6N_c^2}$	$-\frac{(3N_c+7)}{24N_c^2}$
$^4 8_{\frac{3}{2}}$	$\frac{1}{2N_c}$	$\frac{1}{3N_c}$	$-\frac{(3N_c+1)}{8N_c^2}$	$-\frac{(3N_c+1)}{3N_c^2}$	$-\frac{(3N_c+1)}{4N_c^2}$

TABLE IV: Matrix elements of isospin-singlet octet operators in the **70**-plet.

	B_1	B_2	B_3
	t_8	T_8^c	$\frac{10}{N_c} d_{8ab} g_{ia} G_{ib}^c$
$2 8_{\frac{1}{2}}^{-}, 2 8_{\frac{3}{2}}^{-}$	$\frac{N_c^3 + (7S+8I^2)N_c^2 - 3(4S+8I^2-1)N_c + 9S}{2\sqrt{3}N_c(N_c-1)(N_c+3)}$	$\frac{N_c^4 + (3S+1)N_c^3 - (S+8I^2+3)N_c^2 + 3(S+8I^2-1)N_c - 9S}{2\sqrt{3}N_c(N_c-1)(N_c+3)}$	$\frac{3N_c^3 + (13S+8I^2-3)N_c^2 - (31S+44I^2+12)N_c + 6(S+14I^2)}{-\frac{24}{5}\sqrt{3}N_c^2(N_c-1)}$
$4 8_{\frac{1}{2}}^{-}, 4 8_{\frac{3}{2}}^{-}$	$\frac{N_c + S - 4I^2}{2\sqrt{3}(N_c-1)}$	$\frac{N_c^2 + (3S-2)N_c + 4(I^2-S)}{2\sqrt{3}(N_c-1)}$	$\frac{3N_c^2 + (7S-4I^2+3)N_c - (S+20I^2)}{-\frac{24}{5}\sqrt{3}N_c(N_c-1)}$
$2 8_{\frac{1}{2}}^{-} - 4 8_{\frac{1}{2}}^{-}$	0	0	0
$2 8_{\frac{3}{2}}^{-} - 4 8_{\frac{3}{2}}^{-}$	$\frac{(3-N_c)}{\sqrt{3}(N_c+3)}$	$\frac{(N_c+5)(N_c-3)}{2\sqrt{3}(N_c+3)}$	0
$2 1_{\frac{1}{2}}^{-}, 2 1_{\frac{3}{2}}^{-}$	$-\frac{3(N_c-1)}{2\sqrt{3}N_c(N_c+3)}$	$-\frac{3(N_c-1)}{2\sqrt{3}N_c(N_c+3)}$	$\frac{5(3N_c+1)}{16N_c\sqrt{N_c}}$
$2 8_{\frac{1}{2}}^{-} - 2 1_{\frac{1}{2}}^{-}$	0	0	0
$2 8_{\frac{3}{2}}^{-} - 2 1_{\frac{3}{2}}^{-}$	$\frac{N_c + 8S + 5}{2\sqrt{3}(N_c+5)}$	$\frac{N_c^2 + (3S+4)N_c + 7S-5}{2\sqrt{3}(N_c+5)}$	$-\frac{3N_c^2 + 14(S+1)N_c + 22S-5}{\frac{24}{5}\sqrt{3}N_c(N_c+5)}$
$4 8_{\frac{1}{2}}^{-} - 2 1_{\frac{1}{2}}^{-}$	$-\sqrt{\frac{2}{3}}\sqrt{\frac{N_c+3}{N_c(N_c-1)(N_c+5)}}$	$\sqrt{\frac{2}{3}}\sqrt{\frac{N_c+3}{N_c(N_c-1)(N_c+5)}}$	$\frac{5(N_c+2)}{6\sqrt{6}N_c}\sqrt{\frac{N_c+3}{N_c(N_c-1)(N_c+5)}}$
$4 8_{\frac{3}{2}}^{-} - 2 1_{\frac{3}{2}}^{-}$	0	0	0

TABLE V: Matrix elements of the isospin-singlet octet operator B_4 in the **70**-plet.

B_4	
$3l_i$	g_{is}
$28_{\frac{1}{2}}$	$-\frac{N_c^3+(10S+14I^2-3)N_c^2-3(7S+8I^2)N_c+9(S+2I^2)}{\sqrt{3}N_c(N_c-1)(N_c+3)}$
$48_{\frac{1}{2}}$	$-\frac{5(N_c+S-4I^2)}{4\sqrt{3}(N_c-1)}$
$28_{\frac{1}{2}}-^48_{\frac{1}{2}}$	$-\frac{N_c+S-4I^2}{2\sqrt{6}(N_c-1)}\sqrt{1+\frac{3}{N_c}}$
$21_{\frac{1}{2}}$	$\frac{\sqrt{3}(N_c-3)}{(N_c+3)}$
$28_{\frac{1}{2}}-^21_{\frac{1}{2}}$	$\frac{9(N_c-1)}{2(N_c+3)\sqrt{N_c}}$
$48_{\frac{1}{2}}-^21_{\frac{1}{2}}$	0
$210_{\frac{1}{2}}$	$\frac{N_c+8S+5}{2\sqrt{3}(N_c+5)}$
$28_{\frac{1}{2}}-^210_{\frac{1}{2}}$	$-\sqrt{\frac{2}{3}}\sqrt{\frac{N_c+3}{N_c(N_c-1)(N_c+5)}}$
$48_{\frac{1}{2}}-^210_{\frac{1}{2}}$	$\frac{4}{\sqrt{3}}\frac{1}{\sqrt{(N_c-1)(N_c+5)}}$
$28_{\frac{3}{2}}$	$\frac{N_c^3+(10S+14I^2-3)N_c^2-3(7S+8I^2)N_c+9(S+2I^2)}{2\sqrt{3}N_c(N_c-1)(N_c+3)}$
$48_{\frac{3}{2}}$	$-\frac{N_c+S-4I^2}{2\sqrt{3}(N_c-1)}$
$28_{\frac{3}{2}}-^48_{\frac{3}{2}}$	$-\sqrt{\frac{5}{3}}\frac{N_c+S-4I^2}{4(N_c-1)}\sqrt{1+\frac{3}{N_c}}$
$21_{\frac{3}{2}}$	$-\frac{\sqrt{3}(N_c-3)}{2(N_c+3)}$
$28_{\frac{3}{2}}-^21_{\frac{3}{2}}$	$-\frac{9(N_c-1)}{4(N_c+3)\sqrt{N_c}}$
$48_{\frac{3}{2}}-^21_{\frac{3}{2}}$	0
$210_{\frac{3}{2}}$	$-\frac{N_c+8S+5}{4\sqrt{3}(N_c+5)}$
$28_{\frac{3}{2}}-^210_{\frac{3}{2}}$	$\sqrt{\frac{N_c+3}{6N_c(N_c-1)(N_c+5)}}$
$48_{\frac{3}{2}}-^210_{\frac{3}{2}}$	$\frac{2\sqrt{10}}{\sqrt{3(N_c-1)(N_c+5)}}$
$48_{\frac{5}{2}}$	$\frac{\sqrt{3}(N_c+S-4I^2)}{4(N_c-1)}$

TABLE VI: Masses and spin-flavor content as predicted by the $1/N_c$ expansion[11] respectively depicted in the third and last four columns . The empirical values of the masses for all states with a status of three or more stars in [36] are shown in the second column and the results from the quark model calculation of Isgur and Karl [31] are shown in the fourth column.

State	Masses [MeV]			Spin-flavor content			
	Expt.	Large N_c	QM	2_1	2_8	4_8	$^2_{10}$
$N_{1/2}$	1538 ± 18	1541	1490		0.82	0.57	
$\Lambda_{1/2}$	1670 ± 10	1667	1650	-0.21	0.90	0.37	
$\Sigma_{1/2}$	(1620)	1637	1650		0.52	0.81	0.27
$\Xi_{1/2}$		1779	1780		0.85	0.44	0.29
$N_{3/2}$	1523 ± 8	1532	1535		-0.99	0.10	
$\Lambda_{3/2}$	1690 ± 5	1676	1690	0.18	-0.98	0.09	
$\Sigma_{3/2}$	1675 ± 10	1667	1675		-0.98	-0.01	-0.19
$\Xi_{3/2}$	1823 ± 5	1815	1800		-0.98	0.03	-0.19
$N'_{1/2}$	1660 ± 20	1660	1655		-0.57	0.82	
$\Lambda'_{1/2}$	1785 ± 65	1806	1800	0.10	-0.38	0.92	
$\Sigma'_{1/2}$	1765 ± 35	1755	1750		-0.83	0.54	0.17
$\Xi'_{1/2}$		1927	1900		-0.46	0.87	0.18
$N'_{3/2}$	1700 ± 50	1699	1745		-0.10	-0.99	
$\Lambda'_{3/2}$		1864	1880	0.01	-0.09	-0.99	
$\Sigma'_{3/2}$		1769	1815		0.01	(-0.57)	(-0.82)
$\Xi'_{3/2}$		1980	1985		-0.02	(-0.57)	(-0.82)
$N_{5/2}$	1678 ± 8	1671	1670			1.00	
$\Lambda_{5/2}$	1820 ± 10	1836	1815			1.00	
$\Sigma_{5/2}$	1775 ± 5	1784	1760			1.00	
$\Xi_{5/2}$		1974	1930			1.00	
$\Delta_{1/2}$	1645 ± 30	1645	1685				1.00
$\Sigma''_{1/2}$		1784	1810		-0.14	-0.31	0.94
$\Xi''_{1/2}$		1922	1930		-0.14	-0.31	0.94
$\Omega_{1/2}$		2061	2020				1.00
$\Delta_{3/2}$	1720 ± 50	1720	1685				1.00
$\Sigma''_{3/2}$		1847	1805		-0.19	(-0.80)	(0.57)
$\Xi''_{3/2}$		1973	1920		-0.19	(-0.80)	(0.57)
$\Omega_{3/2}$		2100	2020				1.00
$\Lambda''_{1/2}$	1407 ± 4	1407	1490	0.97	0.23	0.04	
$\Lambda''_{3/2}$	1520 ± 1	1520	1490	0.98	0.18	-0.01	

TABLE VII: Partial contributions to masses (in MeV) by singlet operators. The first two rows contain the off diagonal contributions.

	$c_1 O_1$	$c_2 O_2$	$c_3 O_3$	$c_4 O_4$	$c_5 O_5$	$c_6 O_6$	$c_7 O_7$	$c_8 O_8$	$c_9 O_9$	$c_{10} O_{10}$	$c_{11} O_{11}$	O_{total}
$N_{1/2} - N'_{1/2}$	0	-17	-57	12	8	0	0	1	5	0	-8	-56
$N_{3/2} - N'_{3/2}$	0	-27	18	19	13	0	0	0	7	0	-13	17
$N_{1/2}$	1347	-17	0	86	-16	160	27	0	-25	16	4	1580
$N_{3/2}$	1347	9	0	-43	8	160	27	0	13	16	-2	1534
$N'_{1/2}$	1347	-43	-40	76	-41	320	-27	2	16	31	-21	1621
$N'_{3/2}$	1347	-17	32	31	-16	320	-27	-1	7	31	-8	1698
$N_{5/2}$	1347	26	-8	-46	25	320	-27	0	-10	31	12	1671
$\Delta_{1/2}$	1347	17	0	-49	-33	320	53	0	21	-25	-7	1645
$\Delta_{3/2}$	1347	-9	0	24	16	320	53	0	-11	-25	3	1720
$\Lambda''_{1/2}$	1347	-52	0	0	0	0	0	0	0	0	0	1295
$\Lambda''_{3/2}$	1347	26	0	0	0	0	0	0	0	0	0	1373

TABLE VIII: Partial fits of non-strange masses and the singlet Λ masses using subsets of singlet operators. The mixing angles are predicted except for the last row. The last row results from solving for the coefficients c_1, \dots, c_{11} with the eleven inputs given by the seven non-strange masses, the two singlet Λ masses and the two leading order mixing angles. The singlet Λ masses are corrected by a shift of 135 MeV to include $SU(3)$ breaking.

c_1	c_2	c_3	c_4	c_5	c_6	c_7	c_8	c_9	c_{10}	c_{11}	χ^2/dof	θ_1	θ_2
517	-	-	-	-	-	-	-	-	-	-	99.1	0	0.
448	-	-	-	-	508	-	-	-	-	-	5.4	0	0.
447	-	40	-	-	514	-	-	-	-	-	6.1	0.12	3.11
453	64	-	51	-	474	-	-	-	-	-	2.2	0.13	0.15
454	69	68	63	-	481	-	-	-	-	-	2.3	0.40	0.08
451	76	113	114	79	507	-122	-	-	-	-	1.1	0.72	3.08
449	75	123	109	77	487	-125	9	16	-64	18	exact	0.61	3.04

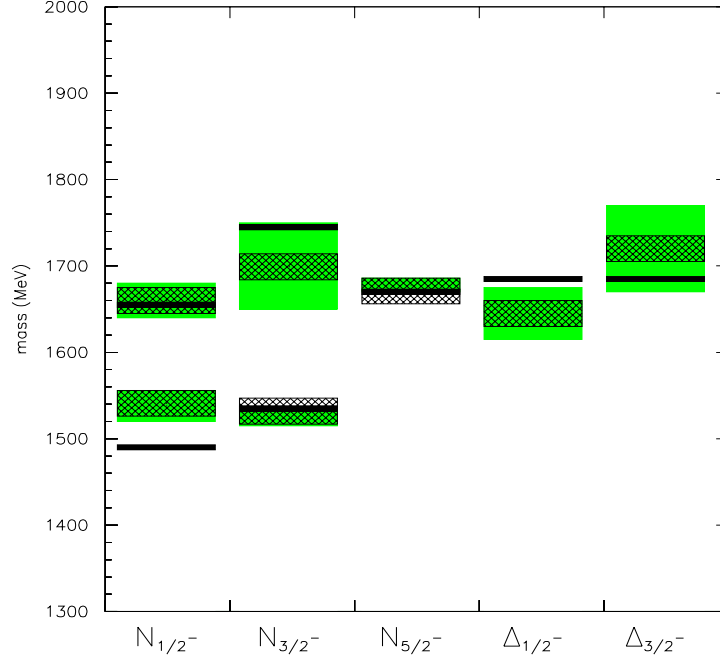


FIG. 1: Non-strange baryon masses: the shaded boxes correspond to the experimental data[36], the black lines are the Isgur-Karl quark model predictions[31] and the hatched boxes are the $1/N_c$ results[11].

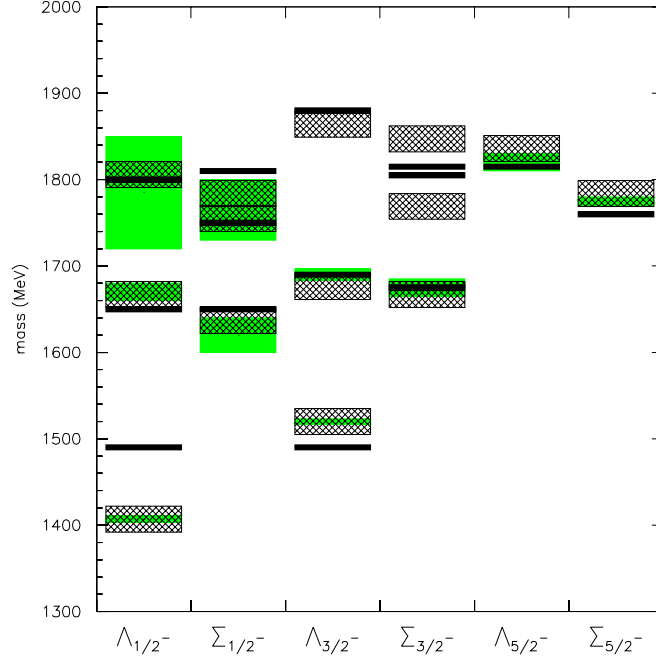


FIG. 2: Masses of baryons with one unit of strangeness.

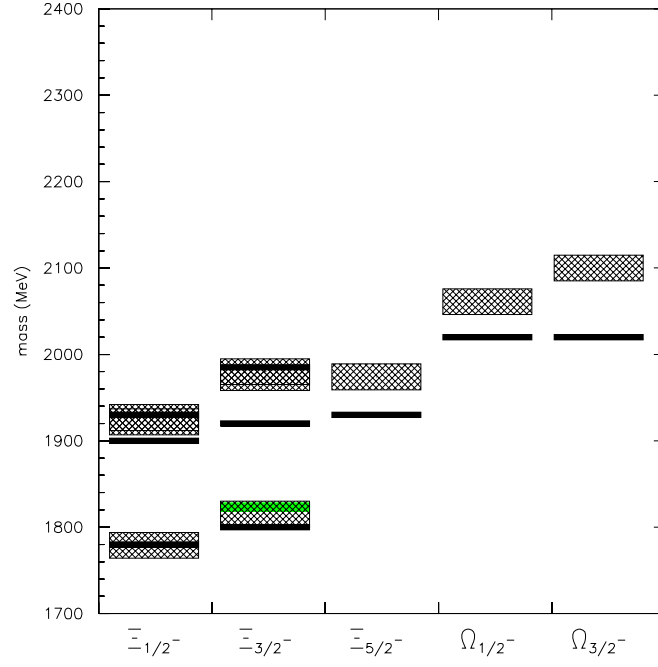


FIG. 3: Masses of baryons with two and three units of strangeness.